STATISTICAL DECISION THEORY AS A GUIDE TO INFORMATION PROCESSING

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STATISTICAL DECISION THEORY AS A GUIDE TO INFORMATION PROCESSING

by

Harvey M. Wagner*

Economists, statisticians, and practitioners of operations research frequently meet nearly identical problems in their respective studies. Once the similarities are recognized, the solutions advanced by one group of professionals often turn out to be useful to others in different disciplines. The belief expressed here is that statistical decision theory provides both an enlightening and a unifying approach to problems concerned with decision making in the face of uncertainty. As will be pointed out subsequently, statistical decision theory is by no means the last word on such problems--at least at its present state of development--but the approach seems to ask the right questions and accurately pinpoints the areas of difficulty.

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* This paper was written while I was affiliated with Stanford University and presented at the Data Processing and Management Information Conference, Massachusetts Institute of Technology, July 15-19, 1957. The author owes more than the usual debt of gratitude to Professors Herman Chernoff and Lincoln Moses, Stanford University, for permission to read their forthcoming book on decision theory.

INTRODUCTION

The advance which decision theory makes over previous methods in mathematical statistics is that the economic consequences of an action are explicitly taken into account. In other words, the theory goes beyond statements about probabilities of making various errors, and incorporates both the relative losses from such errors as well as the costs of processing information in order to reduce the likelihood of mistakes. One important consequence claimed by decision theorists is that by such analysis it is possible to unify various subfields in statistics into a single conceptual framework. For the moment we shall refrain from stating the alleged disadvantages of the theory.

The general problem of decision making, whether studied by a statistician, an economist, or an operations researcher, can conveniently be stated as follows: The decision maker has to choose some course of action out of several open to him.[1] Such an action may pertain to an existing state of affairs or to future events; in any case, the decision maker does not know what the true state really is, and hence he has to choose an action under conditions of uncertainty. The economic consequences of the situation are a joint function of the action

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[1]We shall not be concerned with the organizational or team problems of decision making. We assume that the individual, team, organization, etc., all have identical goals.

taken and the true but, at present, unknown state of affairs. It is useful to think of this situation as a game played by Nature, who chooses the underlying state, and the Statistician (or the decision maker), who selects an action. Usually the Statistician, by means of relatively costly data processing, is able to obtain some information about the strategy Nature has selected. The Statistician must balance the costs of data processing with the costs of making mistakes at a frequency which could potentially be lowered if more information were available. The data conceivably available to the Statistician may or may not be able to give complete information as to Nature's strategy.

The above formulation applies easily to the case of a management group making some decision about the company's sales, production, or investment policies by "sampling" information. The cost of sampling of course may include the use of an electronic computer as well as the expense of collecting data. Consequently a wide variety of data processing problems may potentially be handled by decision theory techniques.

OUTLINE OF A DECISION THEORY PROBLEM

Statistical decision theory, not unlike schools of thought in economics, mathematics, or philosophy, is based on a system of axioms. These postulates are far from inconsequential, but space limitations prohibit a lengthy discussion of the axioms. Briefly, their main implication is that it is possible to assign numerical values to the joint result of the Statistician's and Nature's strategies; these numerical values are what

we have been calling the "economic consequences" of the final situation from the point of view of the Statistician.[2] Further, given Nature's choice, if one action results in a numerical value of 10, say, and another action results in a numerical value of 20, then the combined "action" of flipping a fair coin, so that on heads the first action is taken, and on tails the second action is taken, has the numerical value of the arithmetical average $\frac{1}{2} \times 10 + \frac{1}{2} \times 20 = 15$. In most elementary presentations of the theory of games, the numerical value is usually assumed to be the monetary consequence or payoff of the situation. Such an additional assumption may or may not be tenable in a particular case; but in any event, decision theory assumes that some numerical indicator of preference for various situations is available, and that it is meaningful to take probability averages of these numbers in evaluating the relative merit of different combinations of uncertain outcomes.

For expositional simplicity assume that Nature and the Statistician have a finite number of simple alternatives: Nature's choices are $N_1, N_2, \ldots, N_i, \ldots$ and the Statistician's actions are $a_1, a_2, \ldots, a_j, \ldots$. The Statistician's numerical indicator of the outcome of Nature's selecting N_i and his taking action a_j is denoted as $u(N_i, a_j)$. The entire set of consequences can be displayed in matrix form, Exhibit 1.

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[2]In the technical literature the numerical indicator is called the Statistician's utility function.

Suppose that the Statistician has the opportunity of performing a single costless experiment. The experiment may be complicated and may offer a variety of bits of information, but assume that the outcome of the experiment can be summarized by a "vector" symbol z_k, and that there are only a finite number of z_k. For example, one experiment might be a yes-no questionnaire; in this event z_k would be a "vector" of information yielding the number of yes answers to the first question, to the second question, ..., to the n-th question.

By assumption, the data z_k are related to N_i. More precisely, suppose that, given any N_i , the probability of observing z_k is known, which is denoted as the "conditional" probability $p(z_k | N_i)$. Once again the conditional probabilities can be arrayed by means of a table, Exhibit 2. Each row in the matrix indicates the conditional probability of observing every z_k given that N_i is the true state of nature.

Next we define the notion of a simple strategy for the Statistician. Recall that the Statistician may observe any z_k and accordingly take any action a_j. Conceptually all possible simple strategies available can be formulated by listing all combinations of actions associated with observations, Exhibit 3. Each row in the matrix is a simple strategy, which specifies the action to be taken if a z_k is observed. Altogether the number of simple strategies are:

$$(\text{number of actions})^{\text{number of possible observations}} .$$

For example, if there are two actions and five possible observations, then there are $2^5 = 32$ simple strategies.

From Exhibits 2 and 3 we are able to construct a matrix for each strategy s_h, which yields the probability $p(a_j | N_i, s_h)$ of taking a particular action a_j, given Nature's N_i and strategy s_h, Exhibit 4.

Finally Exhibits 4 and 1 are combined to produce a table showing the <u>expected</u> or average numerical values for each pair of strategies. Since for a particular strategy Exhibit 4 gives the probability of taking an action for each state of nature, and since Exhibit 1 contains the numerical consequences associated with each action and state of nature, we average the numerical outcomes and enter them in Exhibit 5 as $U(N_i, s_h) = \sum_j p(a_j | N_i, s_h)\, u(N_i, a_j)$.

Exhibit 5 completely embodies the problem as defined. It shows all the simple strategies open to the Statistician and to Nature. In addition to these simple strategies, each player can also elect to "randomize" between the simple strategies, i.e., to select each simple strategy according to a certain probability.

It is now appropriate to discuss the difficult topic of what is a good strategy for the Statistician. It should be stated at the outset that this is a debatable subject, and various alternative suggestions have been put forth. Only a few of them will be briefly explained; Blackwell and Girshick, and Savage contain more complete treatments.[3] One proposal,

based on a "play safe" notion, is to ignore the data and pick a "minimax" action which protects against the worst possible selection of N_i by Nature. Using either Exhibit 1 or those strategies in Exhibit 5 which ignore z_k (i.e., pick the same a_j for all z_k), determine the worst numerical outcome that may arise with the selection of an a_j; then choose that particular a_j which assures the best out of the "worst" values previously found.

An extension of the above procedure is to use <u>all</u> the strategies in Exhibit 5, and to select the "minimax" from these strategies, now specifically allowing probability mixtures or randomization between strategies, if desirable. The numerical value associated with such a generalized minimax strategy is usually an average value of the $U(N_i, s_h)$ components in Exhibit 5, which in turn are averages derived from Exhibits 1 and 4.

If the Statistician has some a priori information (say, from past relevant experience and data) that Nature selects N_i with probability w_i, then the s_h such that $\sum_i w_i U(N_i, s_h)$ is maximized defines an optimal selection, which is called a Bayes strategy. Even if a priori probabilities about Nature are not known, it is clear that the Statistician should consider only strategies which are at least optimal for some set

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[3]See the Bibliography.

of a priori probabilities. This class of strategies, which here will be called the admissible strategies,[4] is usually considerably narrower than all the simple and mixed strategies implied by Exhibit 5. Interestingly enough, for each possible set of a priori probabilities over the N_i, there is at least one simple strategy s_h which is optimal for the Statistician. In special cases it is possible by appealing to "likelihood ratio" manipulations to determine all the admissible strategies rather easily without the complete enumeration of Exhibits 3 and 5.

The general framework of a statistical game may now be summarized: The Statistician and Nature are the two players, each with certain possible strategies or actions; there is a determinate economic evaluation for the Statistician depending on the outcome of both players' selection of strategies; it is possible for the Statistician to perform experiments and observe information pertaining to Nature's choice of a strategy; out of all possible strategies for the Statistician, attention is confined to the class of admissible strategies, i.e., a strategy which is Bayes for at least some a priori probabilities for Nature. It can be shown that one such

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[4]In mathematical statistics there is a fine distinction between the classes of admissible strategies and of Bayes strategies; further, in special games no admissible strategies may exist. But we shall not be concerned with such technical matters in this paper.

admissible strategy is that associated with the minimax average numerical evaluation, and which may be a good strategy if the Statistician has <u>no</u> a priori information about Nature. In the next section we introduce the cost of sampling, which previously we have ignored.

A CLOSER LOOK AT THE DATA PROCESSING OPERATION

The effects of experimentation will now be more carefully examined to demonstrate an efficient method of extracting information out of the sample data and to delineate the economic consequences of obtaining different amounts of costly information.

Although the conceptual framework advanced above is complete, the extent of enumeration of simple strategies needed to accomplish the analysis, even for ordinary sized problems, may be overwhelming if some shortcuts are not available; furthermore, much of the effort expended in the exhaustive approach is on strategies which turn out to be inadmissible. Fortunately probability theory permits certain important simplifications in the procedures previously outlined.

In the case where no experimental data exist but a priori probabilities for N_i are available, it has been stated that with Exhibit 1 the probability averages over the different $u(N_i, a_j)$ for each action a_j would be calculated, and the correct action would be the one yielding the highest average. If some experimental data z_k do exist, the procedure outlined

for Exhibit 5 may be equivalently performed by using the experimental data to transform the a priori probabilities into what are called a posteriori probabilities; the latter probabilities are then applied to the entries in Exhibit 1 just as the a priori probabilities would be applied in the no experimental data case.

From Exhibit 2 the conditional probability $p(z_k | N_i)$ of observing z_k given N_i is known, and w_i denotes the a priori probability of N_i. As defined by probability theory[5]

$$p(z_k | N_i) = p(z_k \text{ and } N_i) / w_i .$$

In other words, the conditional probability of z_k, given N_i, is equal to the joint probability of <u>both</u> z_k and N_i occuring divided by the a priori probability of N_i. Rearranging terms gives

$$w_i p(z_k | N_i) = p(z_k \text{ and } N_i) .$$

The event of observing z_k is the "sum" of the mutually exclusive and completely exhaustive events of obtaining z_k when N_1 is the true state of nature, z_k when N_2 is the true state of nature, ..., z_k when N_i is the true state of nature, etc. In probability terms

$$\begin{aligned} p(z_k) &= p(z_k \text{ and } N_1) + p(z_k \text{ and } N_2) + \ldots + p(z_k \text{ and } N_i) + \ldots \\ &= w_1 p(z_k | N_1) + w_2 p(z_k | N_2) + \ldots + w_i p(z_k | N_i) + \ldots \end{aligned}$$

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[5] W. J. Dixon and F. J. Massey, Jr., <u>Introduction to Statistical Analysis</u>, McGraw-Hill, New York, 1957, pp. 332-333; A. M. Mood, <u>Introduction to the Theory of Statistics</u>, Mc-Graw-Hill, New York, 1950, pp. 26-30.

Therefore the above formulas are combined to derive the a posteriori probability of N_i given z_k

$$p(N_i|z_k) = p(z_k \text{ and } N_i) / p(z_k), \text{ by definition}$$
$$= w_i p(z_k|N_i) / w_1 p(z_k|N_1) + w_2 p(z_k|N_2) + \ldots + w_i p(z_k|N_i) + \ldots$$
$$= \bar{w}_i$$

which numerically is w_i transformed to an a posteriori probability by multiplying by an appropriate factor that is a function of the actual observed z_k. It can be proved that the Bayes procedure as outlined with Exhibits 1-5 is equivalent to the procedure of applying the a posteriori probabilities $\bar{w}_i$ to Exhibit 1. It can also be shown that if successive experiments are performed, e.g., if the information in the vector z_k is actually gotten single experiment by experiment, then the correct procedure is continually to "revise" or to "update" the a posteriori probabilities using the information gained from the new experimental data.

Is the suggested procedure a shortcut? Recall in Exhibit 3 it was necessary to construct a complete listing of every possible strategy; the number of such strategies depended on the number of <u>all possible</u> z_k which could be observed. The shortcut is that a Bayes procedure need only call for certain computations utilizing an <u>actually</u> observed z_k; therefore in practice it is not necessary to list all strategies taking into account any eventuality, but rather to make computations based on the particular result of the experiment. Analogous reasoning applies to the results from a sequence of experiments.

We finally come to the important point of when costly experimentation or data processing should cease and an action be taken. The case of a sequential sampling procedure is discussed here; the simpler case of a fixed sample size plan is examined in the following section. The <u>mathematical</u> condition for the correct stopping place in a sequential game is well defined. The analysis, which is closely related to Bellman's principle of optimality in dynamic programming, is as follows: If a decision is made at the end of some stage of experimentation, the numerical value for the Bayes procedure is found from an average of the a posteriori probabilities and the entries in Exhibit 1. If further experimentation is undertaken, the result will be a random variable, and new a posteriori probabilities will be derived. After an additional observation is processed, a similar calculation is once again made whether further sampling should follow or an action be taken. Because the outcome of an additional observation is a random variable, the decision of what to do next will also be random. The process is repeated until further sampling is uneconomical.

Whenever inspection continues, the cost of making each experiment, reckoned in numerical values consistent with those in Exhibit 1, must be subtracted in order to arrive at the net valuation of further experimentation. Usually more information about Nature's strategy will increase the expected Bayes average valuation. The question is whether the increment in

economic value of more data is offset by the cost of obtaining it. Since the experimental results are random variables, at each stage of the analysis a complicated procedure is needed for computing averages reflecting the valuation of some particular overall sampling strategy. The final decision about a new experiment rests on a comparison of the present a posteriori Bayes average value and the net expected value if another experiment is achieved <u>and the Statistician acts optimally thereafter</u>. As Blackwell and Girshick have demonstrated, in certain special cases (analogous to elementary cases in sequential analysis) the operating procedures for a "sequential statistical game" are fairly simple. In general, a computing procedure for solving such problems is very complex.

AN ILLUSTRATION IN QUALITY CONTROL

An application in the area of quality control will serve to illustrate the decision theory technique.[6] Small lots of a complex assembly item are to be subjected to an acceptance sampling procedure. It is known from experience that the number of defects per item occurs according to a Poisson probability distribution; and for the sake of simplicity, it is postulated here that Nature "produces" lots after selecting

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[6]For a challenging presentation of quality control applied to data processing problems of an accounting nature, see L. L. Vance and J. Neter, <u>Statistical Sampling for Auditors and Accountants</u>, Wiley, New York, 1956.

a Poisson distribution with an average of either 10 or 20 defects per 100 items.[7] In the former case, the lots are acceptable, and in the latter case unacceptable. Exhibit 6 contains the Statistician's payoff matrix. In this example, instead of representing losses as negative numbers employed in a maximizing operation, they are treated as positive numbers, and strategies which minimize loss are to be investigated. It is <u>assumed</u> that these monetary outcomes are good approximations to the Statistician's "utilities."

The minimax strategy for the Statistician, if he does no sampling, is to select a_1 with probability 1/3 and a_2 with probability 2/3. The expected value of the outcome, \$6.66, is then independent of Nature's strategy. If the a priori probability $w_1 = 3/4$ and $w_2 = 1/4$, then a_1 is the optimal action, giving an expected value of \$5.00.

Although the size of a sample is a variable which should be subject to economic analysis in a proposed statistical procedure, assume that for various reasons only 2 items drawn randomly out of the lot are to be inspected. The sample observations will be classified into three categories: $z_1 = 0$ defects, $z_2 = 1$ defect, $z_3 = 2$ or more defects (if two

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[7] We utilize the distinction employed in quality control of defect vs. defective. The latter is defined in terms of the particular number of allowable defects per item.

defects are found in either or both items, inspection ceases). The conditional probabilities for z_k are shown in Exhibit 7.[8]

There are 2 possible actions and 3 possible observations; hence $2^3 = 8$ simple strategies exist, Exhibit 8. Strategy s_4, for example, specifies selecting action a_1 if z_1 occurs, and a_2 otherwise. If N_1 is the true state of nature, then z_1 occurs with probability .82, and consequently action a_1 is taken with probability .82. Exhibit 9 gives the action probabilities for each strategy.[9]

Finally Exhibit 10 combines the previous matrices to give the expected or average losses for each of the strategies. A first glance at Exhibit 10 does not reveal which strategies, if any, are inadmissible; a graphical analysis aids in the process, Figure 1. The expected losses are plotted as two coordinate points with reference to axes for N_1 and N_2. The bottom boundary, which is the lowest convex-to-the-origin boundary defined by strategy points, has the admissible strategies as vertices: s_1, s_2, s_4, s_8.[10]

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[8]If the number of defects in 100 items has a Poisson distribution with an average q, then it is postulated that the number of defects in 2 items is distributed as a Poisson with an average 2q/100.

[9]The mathematician states that each strategy defines a "mapping" from the sample space to the action space.

[10]An alternative delineation of admissible strategies may be found in J. D. Williams, The Compleat Strategyst, McGraw-Hill, 1954, pp. 71-72.

The minimax strategy, found at the intersection of the bottom boundary and a 45° line through the origin, is to select s_4 with probability .46 and s_8 with probability .54. Given a priori probabilities, the corresponding Bayes strategy is found either by applying the probabilities to Exhibit 10, or by finding at which of the admissible strategies it is possible to construct a <u>tangent</u> line with slope $-w_2/w_1$. If $w_1 = 3/4$ and $w_2 = 1/4$, s_4 is the optimal strategy.

If a minimax procedure is to be employed, it has been stated that the expected loss without any data is \$6.66; with data, the minimax expected loss becomes \$6.26. Therefore, it does not pay to take a sample of 2 items unless the sampling cost is less than \$ .40.[11] In the case of $w_1 = 3/4$ and $w_2 = 1/4$, without data the expected loss is \$5.00 , and with data is \$4.70. Hence with this a priori information it pays to inspect two items only if the cost of observation is less than \$ .30. Such considerations are at the heart of selecting a single stage sample size or a sequential sampling procedure.[12]

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[11] This statement must be qualified if there is some value in collecting data, say, for making a future estimate of the a priori probabilities.

[12] As the reader may verify, increasing the sample size has the effect of lowering the boundary line in Figure 1 toward the origin. But the marginal value of successive observations varies with the form of the probability distribution, the sample size, and the a priori probabilities. Hence depending on the aforementioned considerations and data processing costs, it may, for example, pay to take two observations where it would not be economical to take one.

The use of a posteriori probabilities to arrive at a procedure identical to the admissible strategy defined in Exhibit 10 is illustrated with $w_1 = 3/4$ and $w_2 = 1/4$, for which s_4 is optimal.

If z_1 is observed

$\bar{w}_1 = \dfrac{3/4 \times .82}{3/4 \times .82 + 1/4 \times .67} = .79$, $\bar{w}_2 = .21$; upon applying $\bar{w}_1$ and $\bar{w}_2$ to Exhibit 6, a_1 is found optimal.

If z_2 is observed

$\bar{w}_1 = \dfrac{3/4 \times .16}{3/4 \times .16 + 1/4 \times .27} = .64$, $\bar{w}_2 = .36$, and a_2 is optimal.

If z_3 is observed

$\bar{w}_1 = \dfrac{3/4 \times .02}{3/4 \times .02 + 1/4 \times .06} = .50$, $\bar{w}_2 = .50$, and a_2 is optimal.

SUMMARY AND EVALUATION OF THE DECISION THEORY APPROACH

As claimed at the beginning of the paper, the statistical decision theory approach to data processing seems to isolate the crucial points of decision making problems. The outcome of the decision maker's action is a function of not only what he does but what the true state of nature is. In spite of the difficulty of measuring economic consequences of different situations, it seems necessary to assume some sort of economic evaluation in order to arrive at any semblance of rationality in a systematic approach to decision making. The decision theory technique "automatically" weighs the different economic considerations involved in taking actions and gathering information.

In closing, some of the serious drawbacks which appear in the suggested approach should be discussed. It is very important to realize that the limitations cited below may very well apply to <u>any</u> systematic method. Criticisms have been made at several levels of analysis. One set of criticisms concerns (a) the possibility of setting up a meaningful game in the first place, and (b) the feasability of placing economic evaluations on different outcomes. The latter is partly answered by the reply that <u>any</u> statistical procedure has in it either an implicit or an explicit economic evaluation of outcomes. It is more realistic (and courageous!) to make such considerations explicit rather than implicit. The former argument, for example, questions the notions and assumptions involved in Exhibit 2. Whether the requisite probability information is available is a factual matter to be determined for various situations. When such information is lacking, one should immediately be on guard in judging alternative approaches.

A second level of difficulty is the amount of mathematical manipulations necessary to obtain an answer. This criticism includes (a) the high level of theoretical mathematics demanded to analyze a statistical game, (b) as a consequence, the concentrated effort needed to attain new theoretical answers, and (c) the difficult computations required to solve a particular case. Persons familiar with dynamic programming will recognize that, although the latter technique is a very powerful conceptual mode of analysis, even modern-day high speed computers are not

economically able to apply, to particular cases, some of the theoretical results which have been found.[13] Thus decision theory possibly may become a helpful way of taking a first look at a problem or checking an approximate solution.

A third level of difficulty pertains to the selection of a good strategy. Often a priori probabilities of N_i are not known, and correspondingly a Bayes solution is not defined. One answer given to this criticism is that sufficient experimentation will result in a posteriori information "swamping" the a priori assumptions. Such an answer is hardly a convincing defense of the approach. Statisticians, much like economists writing in the area of "new welfare economics,"[14] have often contented themselves with merely characterizing the class of admissible strategies, with the view that this is the class containing all rational strategies. But the practicing statistician will undoubtedly want some further help on choosing one strategy out of this class, and some indication of how his present operating procedures compare with those suggested by the decision theorists.

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[13] S. E. Dreyfus, "Computational Aspects of Dynamic Programming," Operations Research (5), June, 1957, 409-416.

[14] For an elementary presentation, see F. M. Bator, "The Simple Analytics of Welfare Maximization," American Economic Review, March, 1957, pp. 22-59.

In conclusion, the decision theory approach presents a challenging and comprehensive way of looking at data processing problems. Surely any alternative approach should be required to answer the questions posed by decision theory. It remains to be seen whether decision theory has posed all of the essential questions, and furthermore whether it will be able to answer those queries which already have been formulated.

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Possible Actions

States of Nature		a_1	a_2	...	a_j	...
	N_1	$u(N_1, a_1)$	$u(N_1, a_2)$	...	$u(N_1, a_j)$	...
	N_2	$u(N_2, a_1)$	$u(N_2, a_2)$	...	$u(N_2, a_j)$	...
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	N_i	$u(N_i, a_1)$	$u(N_i, a_2)$	...	$u(N_i, a_j)$	...
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Exhibit 1 The Statistician's Economic Evaluation of Possible Actions

Possible Observations

States of Nature		z_1	z_2	...	z_k	...	$\sum$
	N_1	$p(z_1 \mid N_1)$	$p(z_2 \mid N_1)$	...	$p(z_k \mid N_1)$	...	$\sum p(z_k / N_1) = 1$
	N_2	$p(z_1 / N_2)$	$p(z_2 \mid N_2)$	...	$p(z_k / N_2)$	...	$\sum p(z_k \mid N_2) = 1$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	N_i	$p(z_1 \mid N_i)$	$p(z_2 / N_i)$	...	$p(z_k \mid N_i)$	...	$\sum p(z_k / N_i) = 1$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Exhibit 2 Conditional Probabilities $p(z_k / N_i)$

Observations

Strategies		z_1	z_2	z_3	. . .	z_k	. . .	z_n
	s_1	a_{11}	a_{12}	a_{13}	. . .	a_{1k}	. . .	a_{1n}
	s_2	a_{21}	a_{22}	a_{23}	. . .	a_{2k}	. . .	a_{2n}
	s_3	a_{31}	a_{32}	a_{33}	. . .	a_{3k}	. . .	a_{3n}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	s_h	a_{h1}	a_{h2}	a_{h3}	. . .	a_{hk}	. . .	a_{hn}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

a_{hk}: action to be taken using strategy s_h if z_k is observed.

Exhibit 3 Simple Strategies

Actions

States of Nature		a_1	a_2	. . .	a_j	. . .	$\sum$
	N_1	$p(a_1/N_1, s_h)$	$p(a_2/N_1, s_h)$	. . .	$p(a_j\|N_1, s_h)$	. . .	$\sum p(a_j\|N_1, s_h)=1$
	N_2	$p(a_1/N_2, s_h)$	$p(a_2\|N_2, s_h)$	. . .	$p(a_j\|N_2, s_h)$	. . .	$\sum p(a_j/N_2, s_h)=1$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	N_i	$p(a_1\|N_i, s_h)$	$p(a_2\|N_i, s_h)$	. . .	$p(a_j\|N_i, s_h)$	. . .	$\sum p(a_j\|N_i, s_h)=1$
	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Exhibit 4 Action Probabilities for Strategy s_h

Statistician's Strategies

States of Nature

	s_1	s_2	. . .	s_h	. . .
N_1	$U(N_1, s_1)$	$U(N_1, s_2)$	. . .	$U(N_1, s_h)$	. . .
N_2	$U(N_2, s_1)$	$U(N_2, s_2)$	. . .	$U(N_2, s_h)$	. . .
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
N_i	$U(N_i, s_1)$	$U(N_i, s_2)$	. . .	$U(N_i, s_h)$	. . .
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Exhibit 5 <u>Average</u> Economic Evaluation

STATISTICAL DECISION THEORY AS A GUIDE TO INFORMATION PROCESSING

Possible Actions

States of Nature		a_1 Accept Lot	a_2 Reject Lot
	N_1 = 10 defects per 100 items	0	\$10
	N_2 = 20 defects per 100 items	\$20	0

Exhibit 6 Statistician's Losses

Possible Observations

States of Nature	z_1 = 0 defects in 2 items	z_2 = 1 defect in 2 items	z_3 = 2 or more defects in 2 items
N_1	.82	.16	.02
N_2	.67	.27	.06

Exhibit 7 Conditional Probabilities

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Observations

STRATEGIES		z_1	z_2	z_3
	s_1	a_1	a_1	a_1
	s_2	a_1	a_1	a_2
	s_3	a_1	a_2	a_1
	s_4	a_1	a_2	a_2
	s_5	a_2	a_1	a_1
	s_6	a_2	a_1	a_2
	s_7	a_2	a_2	a_1
	s_8	a_2	a_2	a_2

Exhibit 8 Simple Strategies

Strategies

		s_1		s_2		s_3		s_4	
		a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
States of	N_1	1.00	0	.98	.02	.84	.16	.82	.18
Nature	N_2	1.00	0	.94	.06	.73	.27	.67	.33

		s_5		s_6		s_7		s_8	
		a_1	a_2	a_1	a_2	a_1	a_2	a_1	a_2
States of	N_1	.18	.82	.16	.84	.02	.98	0	1.00
Nature	N_2	.33	.67	.27	.73	.06	.94	0	1.00

Exhibit 9 Action Probabilities for Simple Strategies

STATISTICAL DECISION THEORY AS A GUIDE TO INFORMATION PROCESSING

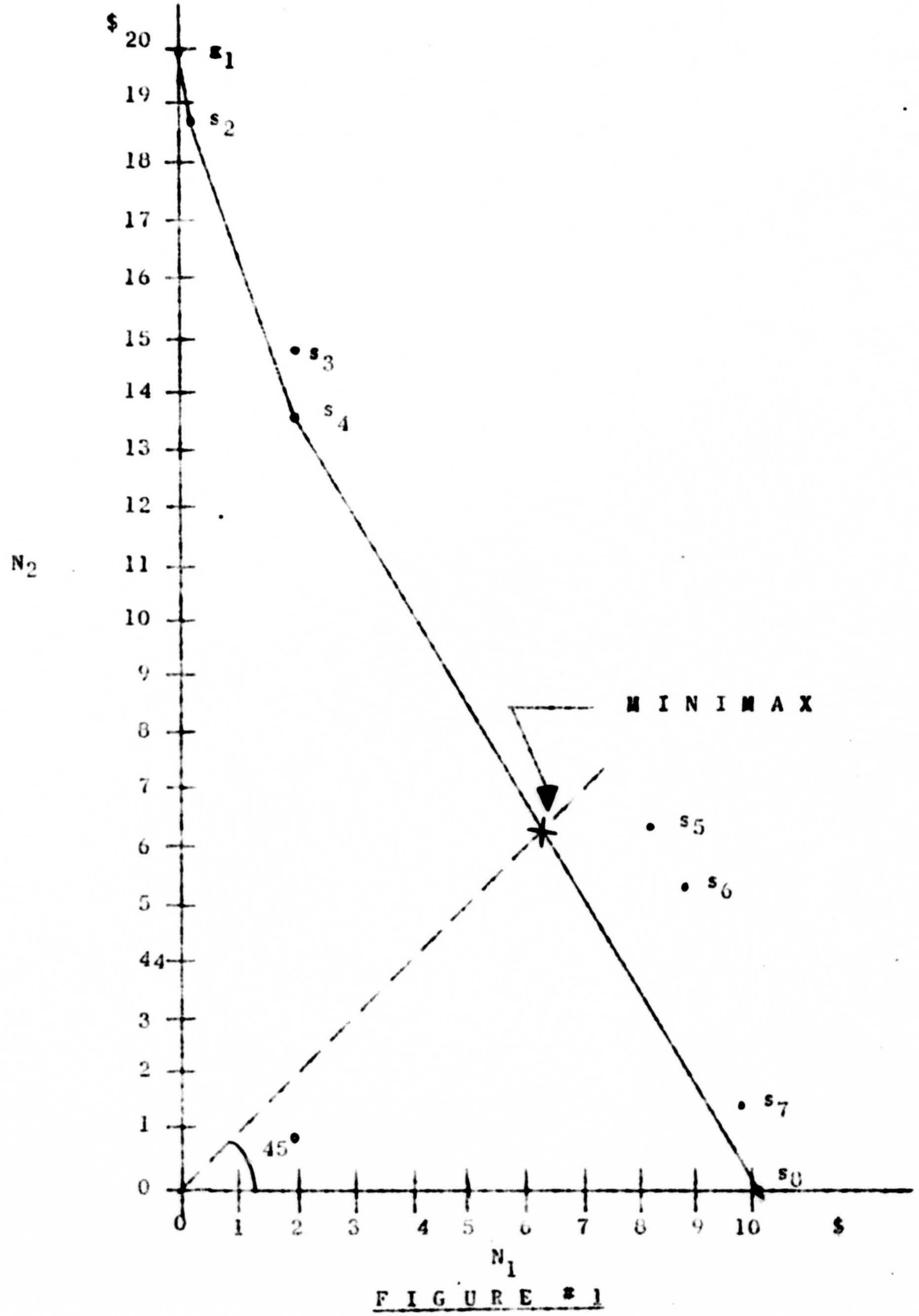
STATISTICIAN'S

STRATEGIES

		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
States of	N_1	0	.20	1.60	1.80	8.20	8.40	9.80	10.00
Nature	N_2	20.00	18.80	14.60	13.40	6.60	5.40	1.20	0

Exhibit 10 Average Economic Evaluation in Dollars

STATISTICAL DECISION THEORY AS A GUIDE TO INFORMATION PROCESSING



F I G U R E # 1